

# Notes for the Wind 3DP Instrument

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# 1 Microchannel Plates

The top-hat analyzer is an instrument which uses two parallel curved plates as a velocity/energy filter. A common version uses a spherical geometry (see Figure 5.1 in *Paschmann and Daly* [1998] for example) with an applied potential between the plates. Curved plate ES analyzers measure  $E/q$  and typically use other instruments to obtain spatial measurements, such as a combination of top-hat with microchannel plates.

A microchannel plate (MCP) consists of a series of small ( $5 \mu\text{m}$  to  $0.25 \text{ mm}$  diameter) holes (or channels) in a thin plate (typically  $0.4\text{-}3.0 \text{ mm}$  thick) made of a conducting material specially fabricated to produce signals similar to a secondary electron analyzer. MCPs are often used in pairs where a cross-sectional cut through two connecting channels creates a v-shaped tube, called a chevron pair. This prevents incident particles from directly impacting the detector behind the plates. When a particle impacts the channel wall, if it has enough energy, it will produce a shower of electrons. The number of electrons per incident particle impact is referred to as the gain of the detector [*Paschmann and Daly*, 1998].

There is a fraction of particles which either strike the pore voids (spaces between channels) or those which do not generate an electron shower/avalanche which are undetected. These factors influence the *quantum efficiency* of the detector. It is important to consider situations when the mean kinetic energy (i.e. temperature) of the background particles is below  $E_{min}$ . Under these conditions, the density and pressure are underestimated but more so for the density, thus the estimate of average kinetic temperature,  $\langle T \rangle = 1/3 \text{ Tr}[\mathbf{P}]/N$ , will be too high. We also find that estimates of bulk velocity magnitudes are too high (direction is okay unless large anisotropies occur). In the opposite scenario ( $\langle KE \rangle > E_{max}$ ), both  $N$  and  $P$  are still underestimated but now  $P$  more so than  $N$ . Thus,  $\langle T \rangle$  is too low and the bulk velocity will be too small as well [*Paschmann and Daly*, 1998].

## 1.1 MCP Efficiency

Before going into too much theory, we should define a physical quantity of an MCP, which is called the open area ratio:

$$f_{OAR} = \left( \frac{\pi\sqrt{3}}{6} \right) \left( \frac{d}{p} \right) \quad (1)$$

where  $d$  is the diameter of the channels and  $p$  is the pitch or center-to-center spacing of the channels. A typical MCP has a channel length-to-diameter ratio of 40:1 and  $f_{OAR} \sim 60\text{-}70\%$ .

### 1.1.1 Goruganthu and Wilson, [1984]

*Goruganthu and Wilson* [1984] calculated the relative electron detection efficiency of a detector consisting of two MCPs in a chevron arrangement with a carbon coated high transmission (94%) copper grid 6 mm in front of the first MCP.

If we let  $T_{max}$  = the maximum emission coefficient,  $\delta(E)$  = secondary emission yield function, and  $E_{max}$  = energy at which efficiency reaches its maximum value [*Bordoni*, 1971], then we have:

$$\epsilon = \frac{1 - e^{-k\delta(E)/\delta_{max}}}{1 - e^k} \quad (2)$$

where  $\delta(E)$  is given by:

$$\delta(E) = \delta_{max} \left( \frac{E}{E_{max}} \right)^{1-\alpha} \left[ \frac{1 - e^{-T_{max}(E/E_{max})^\alpha}}{1 - e^{-T_{max}}} \right] \quad (3)$$

where  $\delta_{max}$  is the maximum value of the secondary emission coefficient,  $k$  is an adjustable parameter that depends upon  $\delta_{max}$  and a complicated probability.

If we assume the distribution of secondary electrons is Poissonian in nature,  $P(n, \delta) = e^{-\delta} \delta^n / n!$  where  $\delta = \delta(E)$  the secondary emission coefficient, then we can show that the extinction probability,  $X$ , of a chain process started by an electron of energy  $E_1$  is given by the smallest root of:

$$X = e^{-\delta(E_1)(1-X)} \quad (4)$$

Therefore, the probability that a process started by an electron with energy  $E_o$  terminates is given by:

$$X_1 = \sum_{n=0}^{\infty} P[n, \delta(E_o)] X^n = e^{-\delta(E_o)(1-X)} \quad (5)$$

where we have used the known relationship:

$$\sum_{n=0}^{\infty} e^{n\lambda} \frac{\lambda^n e^{-\lambda}}{n!} \quad (6)$$

We can simplify the right-hand side of this equation by using Equation 4 to get:

$$\sum_{n=0}^{\infty} P[n, \delta(E_o)] X^n = \sum_{n=0}^{\infty} e^{n[-\delta(E_1)(1-X)]} \frac{\delta^n(E_o) e^{-\delta(E_o)}}{n!} \quad (7a)$$

$$= e^{\delta(E_o)[e^{-\delta(E_1)(1-X)} - 1]} \quad (7b)$$

$$= e^{\delta(E_o)(1-X)} \quad (7c)$$

$$(7d)$$

which leads us to the relationship:

$$e^{\delta(E_o)(1-X)} - 1 = -(1-X) \quad (8)$$

which confirms Equation 4. Now we use:

$$X_1 = e^{\delta(E_o)(1-X)} \quad (9a)$$

$$= \left( e^{-\delta(E_1)(1-X)} \right)^{\delta(E_o)/\delta(E_1)} \quad (9b)$$

$$= X^{\delta(E_o)/\delta(E_1)} \quad (9c)$$

which is Equation 3 in *Bordoni* [1971]. Now if we compute the probability,  $X_2$ , that an incident electron with energy,  $E$ , does not produce any pulses, we find:

$$X_2 = \sum_{n=0}^{\infty} P[n, \delta(E)] X_1^n = e^{-\delta(E)} \cdot e^{\delta(E)X_1} \quad (10)$$

which results in an efficiency of:

$$\epsilon = 1 - X_2 \quad (11a)$$

$$= 1 - e^{-\delta(E)} \cdot e^{\delta(E)X_1} \quad (11b)$$

$$= 1 - e^{-k\delta(E)/\delta_{max}} \quad (11c)$$

where  $k$  is defined by:

$$k = \delta_{max}(1 - X_1) \quad (12)$$

and  $\delta_{max}$  is the maximum value of the secondary emission coefficient.

The values for Equations 2 and 3 used by the Wind/3DP EESA Low detector are:  $T_{max} = 2.283$ ,  $E_{max} = 325$  (eV),  $\alpha = 1.35$ ,  $\delta_{max} = 1.0$ , and  $k = 2.2$ <sup>1</sup>.

### 1.1.2 *Meeks and Siegel, [2008]*

*Meeks and Siegel* [2008] calculated theoretical estimates of the dead time of a radiation detector, where they define the following:

1.  $\tau \equiv$  dead time = The time period when the detector is unable to measure incident particles due to the channel's discharge recovery time, preamp cycle rates, etc. If the count rates are high, then the channel cannot fully recharge causing smaller avalanches, thus less gain which translates to lower counts. The dead time is also defined as the minimum amount of time between two pulses necessary for the detector such that it records two distinct pulses.
2.  $c_r \equiv$  measured count rate
3.  $C_r \equiv$  corrected count rate
4.  $N_t \equiv$  total number of counts
5.  $P(t)\Delta t \equiv$  probability that a detector detects a particle between  $t$  and  $t + \Delta t$
6.  $D_t \equiv$  delay time (assume  $> \tau$ )

From these definitions, we find that:

$$C_r = \frac{c_r}{1 - c_r\tau} \quad (13)$$

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<sup>1</sup>See routine `mcp_efficiency.pro`

and the probability is given by:

$$P(t)\Delta t = C_r e^{-C_r t} \Delta t \quad (14)$$

which has a desirable property that the expectation value of  $t^m$  is given by the simple form:

$$\langle t^m \rangle = \int_0^\infty dt t^m C_r e^{-C_r t} = \frac{m!}{C_r^m} \quad (15)$$

and we find that conveniently,  $\langle t \rangle = 1/C_r$ , which leads to:

$$\frac{\langle t^m \rangle}{\langle t \rangle^m} = m! . \quad (16)$$

If we make a measurement at times  $t_i$  and if our detector is working properly, then discretely we should have:

$$\frac{\sum_{i=0}^{N_t-1} t_i^m / N_t}{\left( \sum_{i=0}^{N_t-1} t_i / N_t \right)^m} = m! . \quad (17)$$

However, we know that there is statistical uncertainty due to finite  $N_t$  and an unknown  $\tau$ . To try and estimate  $\tau$ , we can take our times  $t_i$  and subtract off a delay time,  $D_t$ , until the moments match the expectation values derived from theory, thus we have:

$$\frac{\sum_{i=0}^{N_t-1} (t_i - D_t)^m / N_t}{\left( \sum_{i=0}^{N_t-1} (t_i - D_t) / N_t \right)^m} \quad (18)$$

where we vary  $D_t$  until Equation 17 RHS of Equation 16.

The variance of the numerator in Equation 16 is given by:

$$\sigma_m^2 = \langle t^{2m} \rangle - \langle t^m \rangle^2 \quad (19a)$$

$$\frac{\sigma_m}{\sqrt{N_t}} = \frac{\langle t \rangle^m m!}{\sqrt{N_t}} \sqrt{\frac{2m!}{(m!)^2} - 1} \quad (19b)$$

where Equation 19a is the standard deviation of the numerator in Equation 16. The uncertainty of the denominator is given by:

$$\left( \langle t \rangle \pm \frac{\langle t \rangle}{\sqrt{N_t}} \right)^m \approx \langle t \rangle^m \left( 1 \pm \frac{m}{\sqrt{N_t}} \right) \quad (20)$$

The fractional uncertainties of the numerator and denominator cannot be added in quadrature because the same  $t_i$  appear in both expressions. We can, however, find a fractional uncertainty for Equation 16 by subtracting the squares of the fractional uncertainties of the numerator and denominator and taking the square root to give:

$$\sigma_{tm} = \frac{1}{\sqrt{N_t}} \sqrt{\frac{2m!}{(m!)^2} - (m^2 + 1)} \quad (21)$$

### 1.1.3 *Schecker et al.*, [1992]

We define the following from [*Schecker et al.*, 1992]:

1.  $\tau \equiv$  dead time = The time period when the detector is unable to measure incident particles due to the channel's discharge recovery time, preamp cycle rates, etc. If the count rates are high, then the channel cannot fully recharge causing smaller avalanches, thus less gain which translates to lower counts. The dead time is also defined as the minimum amount of time between two pulses necessary for the detector such that it records two distinct pulses.
2.  $V_{bias} \equiv$  bias voltage of MCP
3.  $V_{sat} \equiv$  potential at which the MCP efficiency no longer depends upon  $V_{bias} \rightarrow V_{bias}$  at which the measured count rate becomes independent of pulse height  $\rightarrow$  pulse height distribution becomes independent of incident particle energy  $\Rightarrow$  small changes in incident particle energy will not affect the impact energy of incident particles
4.  $\delta_e \equiv$  secondary electron emission coefficient
5.  $V_c \equiv$  KE of electron hitting channel wall
6.  $\kappa \equiv$  some constant in  $\delta_e$  equation
7.  $C_r \equiv$  count rate
8.  $C_{cr} \equiv$  critical count rate =  $N_c/\tau$ , where  $N_c$  is the number of channels
9.  $G_e \equiv$  gain function =  $\delta_e^n = (\kappa V_c)^n$ , where  $n$  = number of stages or cascades/avalanches/showers of electrons
10.  $\alpha \equiv$  ratio of channel length, L, to diameter, d
11.  $\phi_e \equiv$  initial energy of an electron emitted perpendicular to the channel wall
12.  $C_{eff} \equiv$  effective capacitance of plate
13.  $I_s(T) \equiv$  stage/strip current
14.  $T \equiv$  temperature of MCP
15.  $\Delta t \equiv N_c/C_r$
16.  $g_f \equiv$  instantaneous scaling factor used to account for preamp, ADC gain, etc.

We start by defining two relationships which were first derived by *Eschard and Manley*, [1971], given by:

$$V_c = \frac{1}{4} \frac{V_{bias}^2}{\alpha^2 \phi_e} \quad (22a)$$

$$n = \frac{4\alpha^2 \phi_e}{V_{bias}} \quad (22b)$$

and using the gain function we have:

$$G_e = \left( \frac{\kappa V_{bias}}{n} \right)^n \quad (23)$$

If we treat the channel as a capacitor which discharges after emitting electrons, then we can define a count-rate and temperature dependent gain function,  $G(\Delta t, T)$ . Therefore, the voltage across the plate,  $V_{bias}$ , will be:

$$\frac{V_{bias}}{n} \rightarrow \frac{V_{bias}}{n} \left[ 1 - e^{-\Delta t / \tau} \right] \quad (24)$$

where  $\tau$  will now act like a recovery time of a capacitor given by:

$$\tau = \frac{V_{bias} C_{eff}}{I_s(T)}. \quad (25)$$

We can simplify this equation by recognizing that:

$$\left| \frac{dn}{dV_{bias}} \right| = \left( \frac{n}{V_{bias}} \right) \ll \left| \frac{dV_c}{dV_{bias}} \right| = \frac{2}{n} \quad (26)$$

which shows that the temperature dependence is dominated by the  $V_c$ -term, which allows us to assume  $n \sim \text{constant}$ . Therefore, we can simplify our gain function estimate by:

$$G(\Delta t, T) = \delta_e^{n-1} \delta_e(t, T) \quad (27a)$$

$$= \left( \frac{\kappa V_{bias}}{n} \right)^{n-1} \left( \frac{\kappa V_{bias}}{n} \right) \left[ 1 - e^{-\Delta t / \tau} \right] \quad (27b)$$

$$= \left( \frac{\kappa V_{bias}}{n} \right)^n \left[ 1 - e^{-I_s \Delta t / (V_{bias} C_{eff})} \right] \quad (27c)$$

where we find that the shape of the  $G(\Delta t, T)$  curve only depends upon  $C_{eff}$  and the parameters  $\kappa$ ,  $\phi_e$ , and  $g_f$  only affect the amplitude.

## 2 Wind 3DP Particle Detector

The Wind/3DP instrument was designed to make full three-dimensional measurements of the distributions of suprathermal electrons and ions in the solar wind. The instrument includes: *...three arrays, each consisting of a pair of double-ended semi-conductor telescopes each with two or three*

*closely sandwiched passivated ion implanted silicon detectors, measure electrons and ions above  $\sim 20$  keV.. and Top-hat symmetrical spherical section electrostatic analyzers with microchannel plate detectors (MCPs) are used to measure ions and electrons from  $\sim 3$  eV to 30 keV.. [Lin et al., 1995].*

The two types of detectors have energy resolutions ranging from  $\Delta E/E \approx 0.3$  for the solid state telescopes (SST) and  $\Delta E/E \approx 0.2$  for the top-hat electrostatic (ES) analyzers. The angular resolutions are  $22.5^\circ \times 36^\circ$  for the SST and  $5.6^\circ$  (near the ecliptic) to  $22.5^\circ$  for the top-hat ES analyzers. The particle detectors can obtain a full  $4\pi$  steradian coverage in one full(half) spin ( $\sim 3$  s) for the SST(top-hat ES analyzers).

## 2.1 Wind 3DP ES Analyzers

The arrays of detectors are mounted on two opposing boomlets, each 0.5 m in length. The top-hat ES analyzers are composed of four separate detectors, each with different geometry factors to cover different ranges of energies. The electron detectors, EESA, and ion detectors, PESA, are each separated into low (L) and high (H) energy detectors. The H and L analyzers contain 24 and 16 discrete anodes, respectively. The anode layout provides a  $5.6^\circ$  polar angle ( $\theta$ ) resolution within  $\pm 22.5^\circ$  of the ecliptic plane,  $11.25^\circ$  to  $\pm 45^\circ$ , and finally  $22.5^\circ$  for  $\theta > 45^\circ$ . The analyzers are swept logarithmically in energy and counters sample at 1024 samples/spin ( $\sim 3$  ms sample period). Thus the analyzers can be set to sample 64 energy samples per sweep at 16 sweeps per spin or 32 energy samples per sweep at 32 sweeps per spin, etc.

Both PESA analyzers have  $R_{1(2)} = 3.75$  cm(4.03 cm) and entrance opening half-angle of  $\sim 19^\circ$ , which gives a  $\Delta R/\langle R \rangle \simeq 0.072$ . They both have  $\Delta E/E = 0.22$  FWHM and  $\Delta\psi = \pm 7^\circ$ . Ions are post-accelerated by a variable grid (initially set to -2500 V) with transmission of 75%. PESA Low is attenuated by a factor of 50 to avoid saturation in the solar wind. PESA Low has an MCP detection efficiency of roughly 50% [Lin et al., 1995]

EESA Low is nearly identical to PESA Low, but the electrons are post accelerated by a +500 V (initially) potential and a single grid attenuation. EESA Low has an MCP detection efficiency of roughly 70% [Lin et al., 1995].

The MCPs are chevron pairs with gain factors of  $\sim 2 \times 10^6$ , roughly 1 mm thick, and a bias angle of  $\sim 8^\circ$ . PESA Low and EESA Low use single  $180^\circ$  half-ring chevron pairs while both EESA High and PESA High use two half-rings to get  $360^\circ$  FOV. The charge pulses produced by the MCPs are collected on anodes and sent to preamplifier-discriminators (AMPTEK A111) and accumulated by 24-bit counters (8C24) [Lin et al., 1995].

The detectors are defined as follows:

1. **EESA-L (EL):** covers electrons from  $\sim 3$  eV to  $\sim 1$  keV<sup>2</sup> with a  $11.25^\circ$  spin phase resolution. EL has a total geometry factor of  $1.26 \times 10^{-2}$  E cm<sup>2</sup>-sr (where E is energy in eV) with a nearly identical  $180^\circ$  field of view (FOV), radial to the spacecraft, to that of PESA-L.
2. **EESA-H (EH):** covers electrons from  $\sim 200$  eV to  $\sim 30$  keV (though typical values vary from a minimum of  $\sim 137$  eV to a maximum of  $\sim 28$  keV) 32 sample energy sweep each  $11.25^\circ$

<sup>2</sup>These values vary from moment structure to moment structure depending on duration of data sampling, spacecraft potential, and whether in burst or survey mode. The typical range is  $\sim 5$  eV to  $\sim 1.11$  keV.

of spacecraft spin. EH has a total geometry factor of  $1.01 \times 10^{-1} \text{ E cm}^2\text{-sr}$ , MCP efficiency of about 70% and grid transmission of about 73%. EH has a  $360^\circ$  planar FOV tangent to the spacecraft surface which can be electrostatically deflected into a cone up to  $\pm 45^\circ$  out of its normal plane.

3. **PESA-L (PL):** covers ions with a 14 sample energy sweep from  $\sim 100 \text{ eV}$  to  $\sim 10 \text{ keV}$  (often energies range from  $\sim 700 \text{ eV}$  to  $\sim 6 \text{ keV}$ ) each  $5.6^\circ$  of spacecraft spin<sup>3</sup>. PL has a total geometry factor of only  $1.62 \times 10^{-4} \text{ E cm}^2\text{-sr}$  but an identical energy-angle response to that of PESA-H.
4. **PESA-H (PH):** covers ions with a 15 sample energy sweep from as low as  $\sim 80 \text{ eV}$  to as high as  $\sim 30 \text{ keV}$  (typical energy range is  $\sim 500 \text{ eV}$  to  $\sim 28 \text{ keV}$ ) each  $11.25^\circ$  of spacecraft spin<sup>4</sup>. PH has a total geometry factor of  $1.49 \times 10^{-2} \text{ E cm}^2\text{-sr}$  with a MCP efficiency of about 50% and grid entrance post transmission of about 75%.

The stats on the entire instrument suite are shown in Table 1.

Table 1: Wind 3DP Instrument Specs

Detector	Particle	Energy Range	Geometry Factor (cm <sup>2</sup> -sr)	FOV (°)	Dynamic Range (eV cm <sup>-2</sup> sr <sup>-1</sup> s <sup>-1</sup> eV <sup>-1</sup> )
EH/FPC	e	100eV - 30keV	0.1 E	360 × 90	$\sim 10^0\text{-}10^8$
EL	e	3eV - 30keV	0.013 E	180 × 14	$\sim 10^2\text{-}10^9$
PH	p	3eV - 30keV	0.015 E	360 × 14	$\sim 10^1\text{-}10^9$
PL	p	3eV - 30keV	0.00016 E	180 × 14	$\sim 10^4\text{-}10^{11}$
SST					
Foil	e	25-400keV	1.7 E	180 × 20	$\sim 10^{-1}\text{-}10^6$
Open	e	20keV - 6MeV	1.7 E	180 × 20	$\sim 10^{-1}\text{-}10^6$

### 3 Distribution Functions

Though it is mathematically easy to arbitrarily define a distribution function (DF) by benignly writing  $f(\vec{x}, \vec{v}, t)$  and then calling it the DF, to do so in data analysis is not so straight forward. The Wind spacecraft has a number of particle detection instruments, of which, we'll focus on the 3DP instrument [*Lin et al.*, 1995].

<sup>3</sup>Note that in survey mode the data structures typically take 25 data points at 14 different energies while in burst mode they take 64 data points at 14 different energies.

<sup>4</sup>Note that PH has multiple data modes where the number of data points per energy bin can be any of the following: 121, 97, 88, 65, or 56.

### 3.1 Particle Data Structures in IDL

Full 3-dimensional particle moments from the Wind/3DP instrument come as data structures in the 3DP software. The list of structure tags often includes:

1. **PROJECT\_NAME**  $\equiv$  'Wind 3D Plasma'
2. **DATA\_NAME**  $\equiv$  'SST Foil' or 'SST Open' or 'Eesa High' or 'Eesa Low' or 'Pesa High' or 'Pesa Low'<sup>5</sup>
3. **UNITS\_NAME**  $\equiv$  'Counts' or 'df' or 'flux' or 'eflux' or 'rate' or 'crate'
4. **UNITS\_PROCEDURE**  $\equiv$  'convert\_so\_units' or 'convert\_sf\_units' or 'convert\_ph\_units' or 'convert\_esa\_units'<sup>6</sup>
5. **TIME**  $\equiv$  Unix time associated with start of data sample (seconds since January 1, 1970)
6. **END\_TIME**  $\equiv$  Unix time associated with end of data sample
7. **TRANGE**  $\equiv$  [TIME,END\_TIME] (s)
8. **INTEG\_T**  $\equiv$  integration time (s) [= END\_TIME - TIME]
9. **DELTA\_T**  $\equiv$  a tag that may be a remnant from a previous mission *i.e.* FAST
10. **MASS**  $\equiv$  particle mass in  $eV/c^2$  but c (the speed of light) is in km/s (*e.g.* for EL or EH, mass =  $5.6856591 \times 10^{-6}$ )
11. **GEOMFACTOR**  $\equiv$  total geometry factor (E cm<sup>2</sup>-sr) reported in original instrument paper [Lin *et al.*, 1995] determined from simulations and physical geometry of the detector
12. **INDEX**  $\equiv$  long integer tag associated with structure
13. **N\_SAMPLES**  $\equiv$  number of 3DP moments in structure (can vary if one desires data for long time periods)
14. **VALID**  $\equiv$  integer value of 1 or 0 depending on whether the structure has useful data or not, respectively
15. **SPIN**  $\equiv$  long integer associated with the spacecraft spin number
16. **NBINS**  $\equiv$  integer value defining the number of data bins
17. **NENERGY**  $\equiv$  integer value defining the number of energy bins
18. **DACCODES**  $\equiv$  integer array for digital to analog converter (DAC) information

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<sup>5</sup>Note that each of these can be appended with 'Burst' if in burst mode.

<sup>6</sup>Note that PH structures incorrectly mark this value as 'convert\_esa\_units' instead of 'convert\_ph\_units'.

19. **VOLTS**  $\equiv$  float array of voltage for DAC (not something user needs to worry about in most cases)
20. **DATA**  $\equiv$  [NENERGY,NBINS]-float array of data points (units defined by UNITS\_NAME)
21. **DDATA**  $\equiv$  [NENERGY,NBINS]-float array of uncertainty in data (user typically needs to supply this)
22. **ENERGY**  $\equiv$  [NENERGY,NBINS]-float array of energy bin values (eV)
23. **DENERGY**  $\equiv$  [NENERGY,NBINS]-float array of differential energies (eV)
24. **PHI**  $\equiv$  [NENERGY,NBINS]-float array of azimuthal angle (deg) covered for each data point at each energy
25. **DPHI**  $\equiv$  [NENERGY,NBINS]-float array of angular resolution of the azimuthal angle (deg)
26. **THETA**  $\equiv$  [NENERGY,NBINS]-float array of poloidal angle (deg) covered for each data point at each energy
27. **DTHETA**  $\equiv$  [NENERGY,NBINS]-float array of angular resolution of the poloidal angle (deg)
28. **BINS**  $\equiv$  [NENERGY,NBINS]-byte array of values that define whether data is good for that particular energy and data point
29. **DT**  $\equiv$  [NENERGY,NBINS]-float array of sample times for each data point ( $= t_{acc}$  defined in Appendix A)
30. **GF**  $\equiv$  [NENERGY,NBINS]-float array of differential geometry factors for each data point which attempt to incorporate instrument response, detector efficiency, and instrument geometries
31. **BKGRATE**  $\equiv$  [NENERGY,NBINS]-float array of background counts
32. **DEADTIME**  $\equiv$  [NENERGY,NBINS]-float array of times where detectors were not taking data (see definition in Appendix A)
33. **DVOLUME**  $\equiv$  [NENERGY,NBINS]-float array of differential volume for each data point  $= \Delta\theta \times \Delta\psi' \times \Delta E_{eff}$  (see Appendix A for definitions)
34. **OMEGA**  $\equiv$  [NBINS]-float array of steradians covered for each data point  $= \Delta\theta \times \Delta\psi'$
35. **MAGF**  $\equiv$  3-Element float array of magnetic field vector (GSE,nT)
36. **VSW**  $\equiv$  3-Element float array of solar wind velocity vector (GSE,km/s)
37. **SC\_POT**  $\equiv$  scalar float of spacecraft potential (eV)

### 3.2 Unit Conversions for Wind 3DP

To convert between different units (for EL, EH, and PL data), a few quantities must be calculated first. Let us assume we start with the units of counts. Let the following quantities be defined:

1.  $E \equiv$  particle kinetic energy (eV) [associated with `dat.ENERGY`]
2.  $N_E \equiv$  number of energy bins [associated with `dat.NENERGY`]
3.  $N_b \equiv$  number of data bins [associated with `dat.NBINS`]
4.  $\delta t \equiv$  sample/accumulation time (s) [associated with `dat.DT`]
5.  $gf \equiv$  differential geometry factor for each data point [associated with `dat.GF*dat.GEOMFACTOR`]
6.  $M_s \equiv$  particle mass of species  $s$  ( $eV/c^2$ ) [associated with `dat.MASS`]
7.  $\tau \equiv$  dead time [associated with `dat.DEADTIME`]
8.  $s_r \equiv$  measured count rate
9.  $d(E,\Omega) \equiv$  the data [associated with `dat.DATA`]
10.  $\delta E \equiv$  the differential energy [associated with `dat.DENERGY`]

We can define the quantity  $s_r$  by:

$$s_r \equiv \frac{d(E,\Omega)}{\delta t} \quad (28)$$

Let,  $\delta t_c$  (unitless) be defined as:

$$\delta t_c \equiv 1 - s_r * \tau \quad (29)$$

where we define values  $\delta t_{c,k} < 0.2$  as bad,  $\Rightarrow$  !VALUES.F\_NAN<sup>7</sup>. The scale factors used to convert from counts to any of the following are:

$$\text{Counts : scale} = 1.0 \quad (30a)$$

$$\text{rate : scale} = \frac{1.0}{\delta t} \quad (30b)$$

$$\text{crate : scale} = \frac{1.0}{\delta t * \delta t_c} \quad (30c)$$

$$\text{eflux : scale} = \frac{1.0}{(\delta t * gf) * \delta t_c} \quad (30d)$$

$$\text{e2flux : scale} = \left( \frac{1.0}{(\delta t * gf) * \delta t_c} \right) * E \quad (30e)$$

$$\text{e3flux : scale} = \left( \frac{1.0}{(\delta t * gf) * \delta t_c} \right) * E^2 \quad (30f)$$

$$\text{flux : scale} = \frac{1.0}{(\delta t * gf * E) * \delta t_c} \quad (30g)$$

$$\text{df : scale} = \left( \frac{1.0}{(\delta t * gf * E^2) * \delta t_c} \right) * \left( \frac{\text{mass}^2}{2.0 \times 10^5} \right) \quad (30h)$$

If we did not start with counts, we could use the following table to correctly convert to the appropriate units:

$$\text{Counts : scale} = \text{scale} * 1.0 \quad (31a)$$

$$\text{rate : scale} = \text{scale} * \delta t \quad (31b)$$

$$\text{crate : scale} = (\text{scale} * \delta t_c) * \delta t \quad (31c)$$

$$\text{eflux : scale} = (\text{scale} * \delta t_c) * (\delta t * gf) \quad (31d)$$

$$\text{flux : scale} = (\text{scale} * \delta t_c) * (\delta t * gf * E) \quad (31e)$$

$$\text{df : scale} = (\text{scale} * \delta t_c) * (\delta t * gf * E^2) * \left( \frac{2.0 \times 10^5}{\text{mass}^2} \right) \quad (31f)$$

where the units associated with each string tag defining each scale factor in Equations 30a-31f can be found in Table 2.

## 4 Distribution Function Calculation

The following sections are an explanation of the routines **moments\_3d.pro** and **moments\_3du.pro**. For reference, see *Curtis et al.* [1989] and *Paschmann and Daly* [1998].

---

<sup>7</sup>Floating point *Not A Number*

Table 2: Units for Wind/3DP Particle Moment Structures

String Tag	Associated Units
Counts	#
Rate	# s <sup>-1</sup>
CRate	# s <sup>-1</sup>
EFlux	eV cm <sup>-2</sup> sr <sup>-1</sup> s <sup>-1</sup> eV <sup>-1</sup>
E2Flux	eV <sup>2</sup> cm <sup>-2</sup> sr <sup>-1</sup> s <sup>-1</sup> eV <sup>-1</sup>
E3Flux	eV <sup>3</sup> cm <sup>-2</sup> sr <sup>-1</sup> s <sup>-1</sup> eV <sup>-1</sup>
Flux	# cm <sup>-2</sup> sr <sup>-1</sup> s <sup>-1</sup> eV <sup>-1</sup>
DF	s <sup>3</sup> cm <sup>-3</sup> km <sup>-3</sup>

#### 4.1 Density (0<sup>th</sup> moment)

To calculate the distribution function, one must first define a weighting factor,  $wt$  (unitless).  $wt$  depends upon the energy, spacecraft potential,  $\phi_{sc}$ , particle charge,  $q$  (in units of fundamental charge), and differential energy. For mathematical and practical purposes,  $wt$  is constrained by defining it as:

$$wt = 0.0 < \left( \frac{E + q \phi_{sc}}{\delta E} + 0.5 \right) < 1.0 \quad (32)$$

We define the total energy at infinity of a particle as:

$$E_{inf} = E + q \phi_{sc} > 0 \quad (33)$$

and this allows us to define a differential velocity,  $\delta v$  ( $\sqrt{eV}$ ), as  $= \sqrt{E_{inf}}$ . If we convert our data into energy flux units, we can define the differential distribution function in the following manner:

$$\delta f \equiv \left( \frac{\delta v}{E} \right) \left( \frac{\delta E}{E} \right) (d(E, \Omega) d\Omega wt) \quad (34)$$

where  $d\Omega$  is the differential volume. At this point,  $\delta f$  has the units of  $eV^{-1/2} cm/s cm^{-3}$  and is a 2-dimensional  $N_E \times N_b$ -array, where the second dimension,  $N_b$ , corresponds to the different data points taken throughout the sample. The number associated with the tag name *MASS* in any 3DP data structure derives from the mass of the particle in units of  $eV/c^2$  and the value of  $c^2$  (speed of light squared) in  $km^2/s^2$  as illustrated the following equation for an electron:

$$m_e = \frac{510,990.6 eV/c^2}{(2.99792458 \times 10^5 km/s)^2} \quad (35a)$$

$$= 5.6967578 \times 10^{-6} eV/(km/s)^2 . \quad (35b)$$

To get  $\delta f$  into useful units, we do the following:

$$\delta f_1 = \delta f \times \frac{M_s}{2} \times 10^{-5} \quad (36)$$

where the factor  $10^5$  is a conversion factor to multiply by  $cm/km$  to get the root of the mass in units of  $eV^{1/2}(cm/s)^{-1}$ , so that now  $\delta f$  has units of  $cm^{-3}$ . Therefore, we can define a particle density of species  $s$  as:

$$N_s = \sum_{i=0}^{N_E-1} \sum_{j=0}^{N_b-1} (\delta f_1)_{i,j} \quad (37)$$

## 4.2 Number Flux (1<sup>st</sup> moment)

A flux, by definition, is a somewhat arbitrary unit of measure when not specified. The reason is that to say flux, one has not defined what the flux is referring to. Flux is defined as the rate of flow of [?], where [?] can be nearly any physical unit. Typical examples include number flux [ $length^{-2} time^{-1}$ ], velocity flux [ $length^{-1} time^{-2}$ ], mass flux [ $mass length^{-2} time^{-1}$ ], energy flux [ $energy length^{-2} time^{-1}$ ], etc. As shown in dimensional analysis, a flux is effectively a density multiplied by a velocity, where the density could be a number, mass, energy, etc. density. It is useful to note that for ES analyzers, the distribution function at energy  $E_i$  and angles  $(\phi_j, \theta_k)$ ,  $f_{i,j,k}$ , is proportional to  $C(E_i, \phi_j, \theta_k) V_{i,j,k}^4$ , where  $C(E_i, \phi_j, \theta_k)$  is the count rate [counts/second] and  $V_{i,j,k}$  is the velocity of the particle. Typically it is assumed that  $f(\mathbf{V})$  is constant over integration volume,  $d^3V$ , where  $d^3V = V^2 dV = V^3 (dV/V)^8$ . Integrals are replaced by summations and the process of calculating the moments is done as usual with the above assumptions.

The number flux calculation requires that we define relative directions/vectors, since flux is a vector quantity. In the spherical coordinate system, we define  $\theta$  (*THETA* in IDL structures) as the polar angle from the XY-plane (thus,  $-\pi/2 \leq \theta \leq \pi/2$ ) and  $\phi$  (*PHI* in IDL structures) as the azimuthal angle from the X-axis (*i.e.* physically this is typically the sun direction for a spinning spacecraft). Therefore, a unit vector  $\hat{r}$  is defined as:

$$\hat{r} \equiv (\cos \theta \cos \phi) \hat{x} + (\cos \theta \sin \phi) \hat{y} + (\sin \theta) \hat{z} \quad (38)$$

The components of the number flux [ $cm^{-2} s^{-1}$ ] are defined by:

$$F_x = \sum_{i=0}^{N_E-1} \sum_{j=0}^{N_b-1} \left[ (\delta f) \cos \phi \cos \theta \left( \frac{E_{inf}}{E} \right) \right]_{i,j} \quad (39a)$$

$$F_y = \sum_{i=0}^{N_E-1} \sum_{j=0}^{N_b-1} \left[ (\delta f) \sin \phi \cos \theta \left( \frac{E_{inf}}{E} \right) \right]_{i,j} \quad (39b)$$

$$F_z = \sum_{i=0}^{N_E-1} \sum_{j=0}^{N_b-1} \left[ (\delta f) \sin \theta \left( \frac{E_{inf}}{E} \right) \right]_{i,j} . \quad (39c)$$

We can see that the units are the same as those for  $[N \mathbf{V}] = [(\# length^{-3}) * (length time^{-1})]$ , which is the definition of a number flux. Note that the bulk velocity,  $\mathbf{V}_s$ , is simply  $= \mathbf{F}_s/N_s$ .

<sup>8</sup>Note:  $dV/V = \text{constant}$  for ES top-hat analyzers since they use  $dE/E = \text{constant}$  in their design

### 4.3 Velocity/Momentum Flux (2<sup>nd</sup> moment)

The next moment of the distribution function is the velocity or momentum flux. The difference is only a factor of mass for nonrelativistic calculations. We start by defining the following:

$$V_o = (\delta f) \left( \frac{E_{inf}^{3/2}}{E} \right) \quad (40)$$

which has units of  $eV^{1/2} cm^{-2} s^{-1}$ . The components of this moment are:

$$dV_{xx} = \sum_{i=0}^{N_E-1} \sum_{j=0}^{N_b-1} [\cos^2 \phi \cos^2 \theta V_o]_{i,j} \quad (41a)$$

$$dV_{yy} = \sum_{i=0}^{N_E-1} \sum_{j=0}^{N_b-1} [\sin^2 \phi \cos^2 \theta V_o]_{i,j} \quad (41b)$$

$$dV_{zz} = \sum_{i=0}^{N_E-1} \sum_{j=0}^{N_b-1} [\sin^2 \theta V_o]_{i,j} \quad (41c)$$

$$dV_{xy} = \sum_{i=0}^{N_E-1} \sum_{j=0}^{N_b-1} [\cos \phi \sin \phi \cos^2 \theta V_o]_{i,j} \quad (41d)$$

$$dV_{xz} = \sum_{i=0}^{N_E-1} \sum_{j=0}^{N_b-1} [\cos \phi \cos \theta \sin \theta V_o]_{i,j} \quad (41e)$$

$$dV_{yz} = \sum_{i=0}^{N_E-1} \sum_{j=0}^{N_b-1} [\sin \phi \cos \theta \sin \theta V_o]_{i,j} \quad (41f)$$

$$(41g)$$

where  $\vec{d}\mathbf{V}_{s,l,m}$  is a symmetric tensor for species  $s$ . We wish to define the velocity flux, which is given by:

$$\vec{V}_{s,l,m} = \vec{d}\mathbf{V}_{s,l,m} \left( \sqrt{\frac{2}{M_s}} \right) \times 10^5 \quad (42)$$

which has units of  $cm^{-1} s^{-2}$ . The momentum flux [ $eV cm^{-3}$ ] is defined by:

$$\vec{p}_{s,l,m} = \vec{V}_{s,l,m} \left( \frac{M_s}{10^{10}} \right) \quad (43)$$

where the factor of  $10^{10}$  is used to convert the units of  $M_s$  to  $eV/(cm/s)^2$ .

The pressure tensor is derived from the momentum, velocity, and number flux, given by:

$$\vec{P}_{s,l,m} = \vec{p}_{s,l,m} - (\mathbf{V}_s \otimes \mathbf{F}_s) \left( \frac{M_s}{10^5} \right) \quad (44)$$

which has the same units as  $\vec{p}_{s,l,m}$ . The *temperature tensor* can be derived from the pressure tensor by dividing by  $N_s$ .

#### 4.4 Heat Flux (3<sup>rd</sup> moment)

To calculate the heat flux [eV km/s  $cm^{-3}$  ( $\simeq 1.602 \times 10^{-7}$  ergs  $cm^{-2} s^{-1}$ )], or kinetic energy flux in the solar wind reference frame, a general treatment requires one to calculate the third moment of the distribution function. In general, the heat flux tensor is defined as:

$$\mathbf{Q}_s = \rho_s \langle (\mathbf{V} - \mathbf{U})(\mathbf{V} - \mathbf{U})(\mathbf{V} - \mathbf{U}) \rangle \quad (45)$$

where  $\rho_s$  is the mass density and  $\mathbf{U}$  is the average bulk velocity given (in general mathematical form) by:

$$\mathbf{U} = \frac{1}{\rho_s} \int d\mathbf{V} f(\vec{\mathbf{x}}, \vec{\mathbf{v}}, t) \mathbf{V}. \quad (46)$$

The pressure tensor is a symmetric tensor, which means  $P_{xy} = P_{yx}$ . If the off-diagonal terms are not zero, then the fluid exhibits shear stresses. The heat flux tensor, in its general form, is a  $3 \times 3 \times 3$ -element array, which, without symmetries, would have 27 distinct elements. However, due to symmetries imposed by assumptions and physical aspects of fluids, we can reduce this tensor to only its symmetric components (10 total). The 10 variations of  $Q_{l,m,n}$  are:  $Q_{x,x,x}$ ,  $Q_{y,y,y}$ ,  $Q_{z,z,z}$ ,  $Q_{x,x,y}$ ,  $Q_{x,x,z}$ ,  $Q_{x,y,z}$ ,  $Q_{y,y,z}$ ,  $Q_{y,z,z}$ ,  $Q_{y,y,y}$ , and  $Q_{z,z,z}$ . The routines MOM\_SUM.PRO and MOM\_TRANSLATE.PRO automatically reduce  $Q_{l,m,n}$  to its symmetric components. Thus, we can find the kinetic energy flux (typically of the electrons) in the solar wind reference frame given by:

$$Q_{l,m,n} = \int d^3v f(\vec{\mathbf{x}}, \vec{\mathbf{v}}, t) v_l v_m v_n. \quad (47)$$

One calculates the moments sequentially and then transforms them into the appropriate coordinate system (GSE in this case). The result is an assumed symmetric tensor which reduces it to a simple rank-2 tensor or  $3 \times 3$  matrix where the sum of the  $i^{th}$  row results in the  $i^{th}$  component of the resultant electron heat flux vector. The above method is the same as the typical definition of the electron heat flux given by:

$$\vec{\mathbf{q}} = \frac{m_e}{2} \int d^3v f(\vec{\mathbf{x}}, \vec{\mathbf{v}}, t) \vec{\mathbf{v}} v^2 \quad (48)$$

where  $m_e$  is the electron mass,  $\vec{\mathbf{v}}$  the velocities, and  $f(\vec{\mathbf{v}}, \vec{\mathbf{x}}, t)$  represents a general form of the distribution function.

## 5 Inflight Calibration

The primary instrument used for bulk solar wind parameters is the PESA Low detector. The PESA Low instrument was equipped with a pinhole attenuator used to reduce the incident flux by a factor of 50 but maintain the same energy-angle response. One of the primary difficulties in calibrating the PESA Low moments resulted from the use of the AMPTEK A111 preamplifier. This preamplifier had a couple of issues which made it difficult to determine its electronic dead time. One of the main difficulties lay in the preamp's response after an input pulse which satisfied the threshold conditions for a count. It turned out that the dead time of the preamp depended on the pulse height distribution of the previous pulse [J.P. McFaddon, *Personal Communication*, July 18, 2011]. Thus,

one is left trying to calibrate the instrument while it is in flight<sup>9</sup>.

The PESA Low instrument is composed of eight anodes which gives the detector a  $5.625^\circ$  poloidal angular resolution within  $\pm 22.5^\circ$  of the spin plane<sup>10</sup>. 64 energy sweeps per spin provide a  $5.625^\circ$  azimuthal angular resolution. The dynamic range of the energy bins (14 total) is adjusted each spin so that  $E_{max}/E_{min} \sim 10$  and  $E_{min} \sim 2/5$  of the peak count rate. This is done so the instrument can track the solar wind and keep a high energy resolution [McFadden *et al.*, 2007; Wüest *et al.*, 2007]. Note that the typical solar wind beam has angular widths comparable to the angular resolution of the PESA Low detector. On board moments are computed from an  $8 \times 8$  array of solid angles(anodes)  $\times$  14 energies for each spin period. Regular PESA Low distributions sent to ground only include  $5 \times 5$  array of solid angles(anodes)<sup>11</sup> in one spin period, once every eight spins or  $\sim 24$  seconds.

## 5.1 Dead Time and Efficiency

We assume that the detector efficiency is constant (or flat) with respect to energy, which is a reasonable assumption for the energy range used. This requires that the preacceleration of the ions into the detector by the plate bias voltage be large enough to make all the ions have impact energies high enough to generate the  $2 \times 10^6$  electrons in the MCP channels. The bias voltage on 3DP was slowly increased during the first four years of the mission to account for *scrubbing*, but has remained relatively constant since<sup>12</sup>. As previously mentioned, the A111 dead time depends upon both the amplitude of the initial and trailing charge pulses, where larger initial pulses can cause larger dead times before a second pulse can be registered.

The peak ion count rate of an ideal detector in response to a drifting Maxwellian is  $\propto N_{io} V_o^4/V_{Tio}^3$ , where  $N_{io}$  is the uncalibrated ion density ( $\text{cm}^{-3}$ ),  $V_o$  is the uncalibrated solar wind bulk flow (km/s), and  $V_{Tio}$  is the uncalibrated ion thermal speed (km/s) [McFadden *et al.*, 2007; Wüest *et al.*, 2007]. If we let  $N_{it}$  be the *true* ion density (values obtained from the SWE key parameters CDF files), then we can say:

$$N_{it} = \frac{\epsilon N_{io}}{1 - \tau R_{max}} \quad (49)$$

where  $\epsilon$  is the absolute detector efficiency,  $\tau$  is the detector dead time correction (seconds), and  $R_{max} = N_{io} V_o^4/(10^6 V_{Tio}^3)$ . The factor  $10^6$  is for unit conversions to make the units of  $R_{max} =$  counts/second.

To calculate the values  $\epsilon$  and  $\tau$ , we can use the following algorithm:

$$\chi^2(\epsilon, \tau) = \sum_{j=0}^{M-1L} \left[ \frac{y_j - Y(\mathbf{x}_j, \mathbf{a})}{\sigma_j} \right]^2 \quad (50)$$

<sup>9</sup>While most missions require in flight calibration due to *scrubbing*, newer missions have used preamplifiers with fixed electronic dead times. Thus, the majority of the calibration deals with the MCP efficiency, since the dead time is a well known constant (except at very high count rates).

<sup>10</sup>A requirement of the Wind SC's attitude is that it maintain a spin axis within  $\pm 0.5^\circ$  of the celestial pole. Thus, the spin plane is effectively the ecliptic plane.

<sup>11</sup>In burst mode, the detector returns the full  $8 \times 8$  array.

<sup>12</sup>The detector is currently set at  $\sim 2.3$ - $2.5$  kV with another  $\sim 1$  kV available if necessary [D. Larson, *Personal Communication*, July 18, 2011]

where  $y_j = (N_{it}/N_{io})_j$ ,  $M$  = number of data points used,  $\sigma = \text{STDDEV}(y,/\text{NAN})*M$ , and the second term is given by:

$$Y(\mathbf{x}_j, \mathbf{a}) = \left( \frac{\varepsilon}{1 - \tau R_{max}} \right)^j. \quad (51)$$

Though we said above that  $(N_{it}/N_{io})$  is a linear function (Equation 49), the use of a  $\chi^2$  distribution provides a way to test the linear dependence of the variables. To construct our  $\chi^2$  distribution, we need to create dummy arrays of possible values for  $\varepsilon$  and  $\tau$  using known physical constraints/limitations. We let  $0.3 \leq \varepsilon \leq 1.2^{13}$  and  $10^{-6} \leq \tau \leq 1.0^{14}$ , where we allow the user to define the number of points in each array. Once the value of  $\chi^2$  has been estimated for all values of  $\varepsilon$  and  $\tau$ , we find the minimum and the corresponding values of  $\varepsilon$  and  $\tau$  are the efficiency and dead time, respectively.

## 5.2 IDL Implementation

The automated routine is called **calc\_pl\_mcp\_eff\_dt.pro** and is located in the `~/wind_3dp_pros/LYNN_PRO/pesa_low_calibration/` directory. The man page provides some useful information and some relevant references.

Let the following definitions be true:

1. nit =  $N_{it}$
2. nio =  $N_{io}$
3. vswm =  $V_o$
4. vti =  $V_{Tio}$
5. rmax =  $R_{max}$
6. and follow the IDL implementation below:

```
slope = nit/nio
gd = N_ELEMENTS(slope)
width = CEIL(1d-2*gd)
; => smooth the slope using autoregressive backcasting
so = remove_noise(slope,NBINS=width)
nn = 100L
sigs = STDDEV(slope,/NAN)*gd
eff = DINDGEN(nn)*( 1.2d0 - 0.3)/(nn - 1L) + 0.3
tau = DINDGEN(nn)*( 1d0 - ALOG10(1d-6))/(nn - 1L) + ALOG10(1d-6)
tau = 1d1^tau
chisq = DBLARR(nn,nn)
FOR j=0L, nn - 1L DO BEGIN
```

<sup>13</sup>The efficiency can be  $>1.0$  here due to unknowns and uncertainties in other parameters, though the true efficiency can never be so. Thus, this value is really an effective efficiency used to modify the detector geometry factor.

<sup>14</sup>The low end estimate results from the 2 MHz clock used, which has a Nyquist period of  $\sim 1 \mu\text{s}$ .

```

FOR k=0L, nn - 1L DO BEGIN
ty = eff[j]/(1d0 - tau[k]*rmax)
num = (so - ty)/sigs[0]
temp = TOTAL(num^2,/NAN,/DOUBLE)
chisq[j,k] = temp[0]
ENDFOR
ENDFOR
; => find minimum
minchisq = MIN(chisq,/NAN,ln)
; => find corresponding element
gmin = ARRAY_INDICES(chisq,ln)
; => define efficiency and dead times
efficiency = eff[gmin[0]]
deadtime = tau[gmin[1]]

```

## A Appendix: Definitions

Let us define the following:

1. FOV  $\equiv$  field of view
2.  $R_{1(2)}$  = hemisphere radius of inner(outer) spherical curved plate analyzer ( $R_2 > R_1$ ), and  $\Delta R = R_2 - R_1$
3.  $E/q \equiv$  kinetic energy per charge of particle traveling at radius  $R = 1/2 \Delta V / \Delta R R_2 R_1 / R$ , where  $\Delta V$  is potential between hemispheres
4.  $k_M \equiv$  analyzer constant =  $(R_2 + R_1) / (2 \Delta R) \simeq 2 (E/q) / \Delta V$
5.  $\Delta E/E \equiv$  acceptance energy range at energy  $E$ , typically held constant so that if  $E/q$  changes,  $\Delta E/q$  changes as well
6. MCP  $\equiv$  microchannel plate
7. channel bias angle  $\equiv$  angle made by MCP channels with respect to normal to plate surface
8. gain factor  $\equiv$  inherent characteristic of channel wall material and a function of electric field intensity inside channel that determines the number of secondary electrons produced per incident particle
9.  $\Delta\psi \equiv$  angular acceptance of a top-hat analyzer [defined by geometry of detector]
10.  $\Delta\theta \equiv$  polar angular resolution defined by discrete anode size and distribution
11.  $\theta \equiv$  polar angle defined by discrete anode positions =  $(n - 0.5) \Delta\theta$ , where  $n =$  zone number for  $n$ th-anode

12.  $\phi \equiv$  azimuthal angle for each  $\theta$
13.  $T_{range} \equiv$  time taken to complete one sweep through the full E/q range
14.  $\Delta\phi \equiv$  azimuthal angular resolution, which for a spinning spacecraft, is controlled by  $T_{range}/T_{spin}$ , where  $T_{spin}$  is the spin period. Typically this ratio is kept to an integer value where  $N_{sweeps} = T_{spin}/T_{range}$ .
15.  $t_{acc} \equiv$  accumulation time = typically a fixed time interval over which data is taken, which corresponds to the azimuthal angular acceptance angle  $\Delta\phi_{acc} \Rightarrow \Delta\psi' \simeq \Delta\psi + \Delta\phi_{acc}$  is the full range of accepted azimuth angles during  $t_{acc}$
16.  $\Delta\psi' \equiv$  angular acceptance angle with spin effects included =  $\Delta\psi + \Delta\phi_{acc} \sin \theta$
17.  $\Delta E_{eff}/q \equiv$  effective passband = range of energies admitted in  $t_{acc}$  which is typically larger than  $\Delta E/q$  due to the change in E/q during  $t_{acc}$  for swept analyzers
18. sweep  $\equiv$  Common practice of how an instrument changes from one energy to another, typically done so that E/q falls by  $\delta E/q$  (typically set to  $\Delta E/q$ ) in  $t_{acc}$ . This is often done logarithmically. If there are 15 energy bins, then there will be 16 steps in one sweep; 15 energy steps and 1 flyback.
19.  $\Delta\phi_{sweep} \equiv$  azimuthal angle through which the spacecraft has swept in  $T_{sweep}$  ( $= T_{range}$ ). Typical instruments constrain parameters so that  $\Delta\phi_{sweep} = \Delta\phi$ .
20.  $\Delta\Omega \equiv$  solid angle resolution =  $\Delta\theta \times \Delta\psi$  (in an *ideal detector* =  $\Delta\theta \times \Delta\phi \sin \theta$ )
21. ES(EM)  $\equiv$  electrostatic(electromagnetic)

## References

- Bordoni, F. (1971), Channel electron multiplier efficiency for 10-1000 eV electrons, *Nucl. Inst. & Meth.*, *97*, 405–+, doi:10.1016/0029-554X(71)90300-4.
- Curtis, D. W., C. W. Carlson, R. P. Lin, G. Paschmann, and H. Reme (1989), On-board data analysis techniques for space plasma particle instruments, *Rev. Sci. Instr.*, *60*, 372–380, doi:10.1063/1.1140441.
- Goruganthu, R. R., and W. G. Wilson (1984), Relative electron detection efficiency of microchannel plates from 0-3 keV, *Rev. Sci. Instr.*, *55*, 2030–2033, doi:10.1063/1.1137709.
- Lin, R. P., et al. (1995), A Three-Dimensional Plasma and Energetic Particle Investigation for the Wind Spacecraft, *Space Science Reviews*, *71*, 125–153, doi:10.1007/BF00751328.
- McFadden, J. P., et al. (2007), In-Flight Instrument Calibration and Performance Verification, *ISSI Sci. Rep. Ser.*, *7*, 277–385.
- Meeks, C., and P. B. Siegel (2008), Dead time correction via the time series, *Amer. J. Phys.*, *76*, 589–590, doi:10.1119/1.2870432.
- Paschmann, G., and P. W. Daly (1998), Analysis Methods for Multi-Spacecraft Data. ISSI Scientific Reports Series SR-001, ESA/ISSI, Vol. 1. ISBN 1608-280X, 1998, *ISSI Sci. Rep. Ser.*, *1*.
- Schecker, J. A., M. M. Schauer, K. Holzscheiter, and M. H. Holzscheiter (1992), The performance of a microchannel plate at cryogenic temperatures and in high magnetic fields, and the detection efficiency for low energy positive hydrogen ions, *Nucl. Inst. & Meth. in Phys. Res. A*, *320*, 556–561, doi:10.1016/0168-9002(92)90950-9.
- Wüest, M., D. S. Evans, and R. von Steiger (2007), *Calibration of Particle Instruments in Space Physics*.